NOVEL PION ELECTROPRODUCTION LOW–ENERGY THEOREMS

V. Bernard[‡,1)], N. Kaiser[⋄,2)], Ulf-G. Meißner[†,3)]

[‡]Centre de Recherches Nucléaires et Université Louis Pasteur de Strasbourg,
Physique Théorique, BP 20Cr, F–67037 Strasbourg Cedex 2, France
[⋄]Technische Universität München, Physik Department T30, James–Franck–Straße,
D–85747 Garching, Germany
[†]Universität Bonn, Institut für Theoretische Kernphysik, Nussallee 14-16,
D–53115 Bonn, Germany

email: [1)]bernard@crnhp4.in2p3.fr, [2)]nkaiser@physik.tu-muenchen.de,
[3)]meissner@pythia.itkp.uni-bonn.de

ABSTRACT

We present novel low–energy theorems for the P–wave multipoles $2M_{1+}+M_{1-}$, $M_{1+}-M_{1-}$, E_{1+} and $L_{1\pm}$ for neutral pion electroproduction off protons. These should be very useful for the analysis of existing or future threshold data.

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Over the last few years, very precise data probing the structure of the nucleon in neutral pion photo– and electroproduction off protons in the threshold region [1] [2] have become available. At present, further data from MAMI at Mainz and NIKHEF at Amsterdam for $\gamma^\star p \to \pi^0 p$ (here, $\gamma^\star$ denotes the virtual photon) are in the process of being analyzed [3] [4]. These are characterized by a small photon four–momentum, $k^2 \simeq -0.1$ GeV^2, and $\Delta W = W - W_{\rm thr}$ of a few MeV, with W the cms energy of the pion–nucleon system in the final state. Even if one restricts oneself to the approximation of retaining only the S– and P–wave multipoles, there are already seven of them. Another complication arises from the fact that certain of these multipoles are much bigger than some others. Therefore, to get a good determination of all multipoles one does not only need a set of very precise data but also model–independent constraints to perform a reliable multipole analysis. We recall that the determination of the electric dipole amplitude E_{0+} in neutral pion photoproduction off protons based on the Mainz data [1] is very sensitive to the treatment of the P–wave multipoles [5] [6] [7] and has only recently been put on a firmer theoretical basis [8] [9]. In this note, we wish to extend this analysis to π^0 electroproduction from protons and show that there exists a set of very useful low–energy theorems for certain P–waves which should be used in the data analysis. To derive these, we explore the strictures of the chiral symmetry of QCD as implemented in an effective field theory, here heavy baryon chiral perturbation theory (HBCHPT) [10] [11]. We note that some of these results were implicetely contained in the calculations presented in ref.[12] making use of relativistic baryon CHPT but have not been made explicit due to the much more complicated arrangement of the chiral expansion in that framework [13]. For a general review, we refer to ref.[14].

To be specific, consider the process $\gamma^\star(k) + p(p_1) \to \pi^0(q) + p(p_2)$ with $k^2 < 0$ and $s = W^2 = (k+p_1)^2 = (q+p_2)^2$ the cms energy squared. In the threshold region, i.e. when the pion three–momentum $\vec{q}$ is close to zero, the transition matrix element $T\cdot\epsilon$ can be written in terms of S– and P–wave multipoles as follows:

$$
\begin{aligned}
\frac{m}{4\pi\sqrt{s}} T\cdot\epsilon = \; & i\vec{\sigma}\cdot\vec{\epsilon}\,[E_{0+} + \hat{q}\cdot\hat{k}\,(3E_{1+}+M_{1+}-M_{1-})] + i\vec{\sigma}\cdot\hat{k}\,\vec{\epsilon}\cdot\hat{q}\,(3E_{1+}-M_{1+}+M_{1-}) \\
& + (\hat{q}\times\hat{k})\cdot\vec{\epsilon}\,(2M_{1+}+M_{1-}) + i\vec{\sigma}\cdot\hat{k}\,\vec{\epsilon}\cdot\hat{k}\,[L_{0+}-E_{0+}+6\,\hat{q}\cdot\hat{k}\,(L_{1+}-E_{1+})] \\
& + i\vec{\sigma}\cdot\hat{q}\,\vec{\epsilon}\cdot\hat{k}\,(L_{1-}-2L_{1+}) \qquad (1)
\end{aligned}
$$

in the πN cm system and in the gauge $\epsilon_0 = 0$. Gauge invariance $k\cdot T = 0$ allows to recover T_0 via $T_0 = \vec{k}\cdot\vec{T}/k_0$. There are two S–wave multipoles, E_{0+} and L_{0+}, and five P–wave multipoles, $M_{1\pm}$, E_{1+} and $L_{1\pm}$. Here E, M and L stand for electric, magnetic and longitudinal, respectively, the first subscript gives the πN angular momentum and the $\pm$ refers to the total angular momentum, $j = l \pm 1/2$. The longitudinal ones, $L_{0+}, L_{1\pm}$ are, of course, specific to electroproduction. All multipoles depend on ω, the pion energy in

the cm system, and on k^2. We will suppress these arguments in what follows. Our aim is to give the threshold expansion for the P–wave multipoles in powers of the dimensionless parameters

$$\mu = \frac{M_\pi}{m}\,, \quad \rho = -\frac{k^2}{M_\pi^2}\,, \tag{2}$$

using HBCHPT. Here, $M_\pi = 134.97$ MeV and $m = 928.27$ MeV are the neutral pion and the proton mass, in order. We will work within the one–loop approximation to order $\mathcal{O}(p^3)$, where p denotes a genuine small momentum. The corresponding chiral counting rules are detailed e.g. in refs.[15] [16]. The effective Lagrangian takes the form

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)}\,, \tag{3}$$

where the superscript '(i)' $(i = 1, 2, 3)$ refers to the chiral dimension. One–loop diagrams start at order p^3. The explicit form of the various terms contributing to the process considered here can be found in ref.[9] together with a determination of the corresponding low–energy constants which appear at orders p^2 and p^3. In fact, only the anomalous magnetic moment of the proton, $\kappa_p = 1.793$, and the constant $b_P = 15.8$ GeV^{-3} will enter our results. The numerical value of b_P can in fact be completely understood by resonance exchange, with a share of 80% from the $\Delta(1232)$ and 20% from vector meson exchange.

Let us first discuss the magnetic multipoles M_{1+} and M_{1-}. From these, one forms the combinations $2M_{1+} + M_{1-}$ and $M_{1+} - M_{1-}$. While the former is completely dominated by the contact term proportional to b_P, the latter shows a rapidly converging expansion in μ [17]. Straightforward calculation gives for the slopes of these particular combinations of the magnetic multipoles at threshold:

$$\frac{1}{|\vec{q}\,|}\,(2M_{1+} + M_{1-})^{\rm thr} = eM_\pi\sqrt{1+\rho}\Big(b_P + \frac{g_{\pi N}}{16\pi m^3}\Big) + \mathcal{O}(\mu^2) \tag{4}$$

$$\begin{aligned}\frac{1}{|\vec{q}\,|}\,(M_{1+} - M_{1-})^{\rm thr} = \frac{e g_{\pi N}}{8\pi m^2}\sqrt{1+\rho}\Big\{1+\kappa_p + \frac{\mu}{4}\Big[-5-2\kappa_p \\ + \frac{g_{\pi N}^2}{8\pi}\Big(\frac{4+3\rho}{1+\rho} - \frac{(2+\rho)^2}{2(1+\rho)^{3/2}}\arccos\frac{-\rho}{2+\rho}\Big)\Big]\Big\} + \mathcal{O}(\mu^2)\end{aligned} \tag{5}$$

with $g_{\pi N} = 13.4$ the strong pion–nucleon coupling constant and $e^2/4\pi = 1/137.036$. The momentum dependence of eq.(4) is entirely given by the square-root $\sqrt{1+\rho}$ arising from $|\vec{k}\,|$ whereas in eq.(5) in addition the function $4 - \Xi_1(\rho)$ enters (cf. eq.(5.2) in ref.[12]). This is shown in fig.1a. We stress that this momentum–dependence is not due to the proton magnetic form factor. Its chiral expansion to order p^3 reads

$$G_M^p(k^2) = 1 + \kappa_p + \frac{g_{\pi N}^2}{32\pi}\mu\Big(2 - \frac{4+\rho}{\sqrt{\rho}}\arctan\frac{\sqrt{\rho}}{2}\Big) + \mathcal{O}(\mu^2)\,, \tag{6}$$

i.e. to the order we are working one is not sensitive to the on–shell electromagnetic form factors. The ρ–dependent function $4-\Xi_1(\rho)$ in eq.(5) can be considered as the leading term in the μ–expansion of the structure function related to the half off–shell photon nucleon vertex (see the discusion in ref.[18]). In the exact chiral limit $\mu=0$ the expression in the curly bracket of eq.(5) becomes, however, equal to the proton magnetic form factor (in the chiral limit), $1+\overset{\circ}{\kappa}_p-(g^2_{\pi N}/64\,\overset{\circ}{m})\sqrt{-k^2}$. Such a behaviour is exactly required by the soft-pion theory. The same features (in the chiral limit) were observed in ref.[20] for double pion electroproduction at threshold.

For the slopes of the small multipoles E_{1+} and $L_{1\pm}$ at threshold, we find

$$\frac{1}{|\vec{q}\,|}\,E^{\rm thr}_{1+}=\frac{eg_{\pi N}\mu}{96\pi m^2}\sqrt{1+\rho}\Big[1+\frac{g^2_{\pi N}}{8\pi}\Big(\frac{8+5\rho}{3(1+\rho)^2}-\frac{(2+\rho)^2}{2(1+\rho)^{5/2}}\arccos\frac{-\rho}{2+\rho}\Big)\Big]+\mathcal{O}(\mu^2) \quad (7)$$

$$\frac{1}{|\vec{q}\,|}\,L^{\rm thr}_{1+}=\frac{eg_{\pi N}\mu}{96\pi m^2}\sqrt{1+\rho}\Big[1+\frac{g^2_{\pi N}}{16\pi}\Big(-\frac{4+\rho}{(1+\rho)^2}+\frac{4-\rho^2}{2(1+\rho)^{5/2}}\arccos\frac{-\rho}{2+\rho}\Big)\Big]+\mathcal{O}(\mu^2) \quad (8)$$

$$\frac{1}{|\vec{q}\,|}\,L^{\rm thr}_{1-}=\frac{eg_{\pi N}\mu}{12\pi m^2}\sqrt{1+\rho}\Big[1+\frac{g^2_{\pi N}}{32\pi}\Big(\frac{2}{1+\rho}-\frac{2+\rho}{(1+\rho)^{3/2}}\arccos\frac{-\rho}{2+\rho}\Big)\Big]+\mathcal{O}(\mu^2)\ . \quad (9)$$

These are shown in fig.1b for $0\le\rho\le 10$. We notice the rather weak momentum dependence of E_{1+} and of L_{1+}, i.e. there is some balance between the square root (which multiplies the Born terms) and the loop contribution as given by the functions in the round brackets of eqs.(7,8). In case of the L_{1-} multipole, this loop–induced momentum dependence overtakes the $\sqrt{1+\rho}$ behaviour. Notice also that in E_{1+} and $L_{1\pm}$ the negative loop term is larger in magnitude than the small positive Born contribution entering at the same chiral order.

Eqs.(4,5,7,8,9) constitute a set of useful low–energy theorems [19] for neutral pion electroproduction off protons and they should be used as constraints in the analysis of the new and upcoming threshold data. The main features of these results are, of course, not totally unexpected. The two magnetic multipoles are dominant, and their k^2–dependence is essentially governed by the $\sqrt{1+\rho}$–factor with an extra weak ρ–dependence for $M_{1+}-M_{1-}$ from the one–loop graphs. The E_{1+} and $L_{1\pm}$ multipoles are small corrections to the dominant $M_{1\pm}$, their momentum–dependence shows a stronger influence of the chiral loops. This latter effect is particularly visible in L_{1-}, cf. fig.1b.

We have not yet made any statement about the two S–wave multipoles E_{0+} and L_{0+}. As detailed in refs.[8] [9] [12], a higher order calculation is mandatory since the corresponding series in μ are slowly converging. At present, a precise experimental determination of these S–wave multipoles is called for making use of the low–energy theorems presented here. However, we remark here that the cusp effect in L_{0+} is expected to be weakened as ρ

increases in comparison to the cusp in the electric dipole amplitude E_{0+}. To be precise, to this order the corresponding imaginary parts read

$$\mathrm{Im} E_{0+}(\omega, k^2) = \frac{e g_{\pi N}}{32\pi^2 F_\pi^2} \mu \sqrt{\omega^2 - \omega_c^2} + \ldots \tag{10}$$

$$\mathrm{Im} L_{0+}(\omega, k^2) = \frac{e g_{\pi N}}{32\pi^2 F_\pi^2} \mu \frac{M_\pi^2}{2M_\pi^2 - k^2} \sqrt{\omega^2 - \omega_c^2} + \ldots \ , \tag{11}$$

with $F_\pi = 93$ MeV the pion decay constant and $\omega_c = 140.11$ MeV corresponding to the opening of the $\pi^+ n$ channel. The strength of the imaginary part above ω_c is intimately connected to the energy variation of the real part below ω_c (the cusp effect) as seen from dispersion theoretical considerations. To leading order, the cusp in E_{0+} is indepedent of k^2 and the corresponding cusp in L_{0+} is suppressed by a factor of $(2+\rho)^{-1}$, i.e. at $\rho = 3$ ($k^2 = -0.055$ GeV^2) it is diminished to 20% of its magnitude in E_{0+}. Consequently, one cannot expect to see such a cusp effect in L_{0+} for typical photon four–momenta of $|k^2| = 0.05 \ldots 0.10$ GeV^2. A further crucial point in π^0 electroproduction is the k^2 dependence of E_{0+}^{thr} and in particular the point where it changes sign. We stress here that the low-energy theorem presented in [12] predicts a stronger increase (in k^2) than conventional pseudovector Born calculations do (due to a particular large loop effect at order p^3, i.e. the additional $\Xi_1(\rho)$ term in eq.(5.1) of ref.[12]). To be specific, the respective slope in k^2 reads

$$\left.\frac{\partial E_{0+}^{\mathrm{thr}}}{\partial k^2}\right|_{k^2=0} = -\frac{e g_{\pi N}}{16\pi m^3}\left(1 + \kappa_p + \frac{4-\pi}{16\pi}\frac{m^2}{F_\pi^2}\right) + \mathcal{O}(\mu) \tag{12}$$

where the last term in the bracket stems from the chiral loops. It amounts to 62% correction to the (conventional) magnetic moment piece. However, to draw decisive conclusions in the S–waves, it seems necessary to perform higher order calculations. We hope to come back to this topic in the near future.

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Figure Captions

Fig.1a The slope of the magnetic multipole combinations $(2M_{1+} + M_{1-})/|\vec{q}|$ (solid line) and $(M_{1+} - M_{1-})/|\vec{q}|$ (dashed line) at threshold versus ρ (in GeV^{-2}).

Fig.1b The slope of the small multipoles $E_{1+}/|\vec{q}|$ (solid line), $L_{1+}/|\vec{q}|$ (dashed line) and $L_{1-}/|\vec{q}|$ (dash–dotted line) at threshold versus ρ (in GeV^{-2}).

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